

Combining FEniCS with Your Favorite Software in C, C++, Fortran, or MATLAB

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1 Introduction

FEniCS is an easy-to-use tool for solving partial differential equations (PDEs) and enables very flexible specifications of PDE problems. However, many scientific problems require (much) more than solving PDEs, and in those cases a FEniCS solver must be coupled to other types of software. This is usually easy and convenient if the FEniCS solver is coded in Python and the other software is either written in Python or easily accessible from Python.

Coupling of FEniCS solvers in Python with MATLAB, Fortran, C, or C++ codes is possible, and in principle straightforward, but there might be a lot of technical details in practice. Many potential FEniCS users already have substantial pieces of software in other more traditional scientific computing languages, and the new solvers they write in FEniCS may need to communicate with this existing and well-tested software. Unfortunately, the world of gluing computer code in very different languages with the aid of tools like F2PY, SWIG, Cython, and Instant is seldom the focal point of a computational scientist. We have therefore written this document to provide some examples and associated detailed explanations on how the mentioned tools can be used to combine FEniCS solvers in Python with other code written in MATLAB, Fortran, C, and C++. We believe that even if the examples are short and limited in complexity, the couplings are technically complicated and broad enough to cover a range of different situations in the real world.

To illustrate the tools and techniques, we focus on four specific case studies:

1. *A flow problem with boundary conditions depending on a system of ODEs.* The ODE solver and the ODE system are implemented in Fortran, and we use F2PY to generate wrapper code such that the ODE solver is accessible from Python.
2. *A flow optimization problem.* The goal here is to compute the optimal placement, permeability and size of a porous medium within a flow field such that certain flow properties are minimized in a given region. A FEniCS flow solver is coupled to sophisticated MATLAB/Octave code for steps in the optimization process (kriging and surrogate modeling).
3. *DOLFIN library-level programming in C++.* A new C++ utility, based on DOLFIN, for fast repeated evaluations of finite element functions on parallel computers is developed. We glue C++ code, DOLFIN, and Python with the aid of the FEniCS JIT compiler Instant.
4. *A parabolic FEniCS solver in Python coupled to a C++ ODE solver at each node.* In this example we illustrate F2PY, SWIG, and Cython as alternative ways for coupling C++ and Python codes.

This tutorial is meant for FEniCS users. For introductions to FEniCS, see the gentle tutorial [4], the paper [6], or the more comprehensive book about the FEniCS project [5].

The present tutorial is found on GitHub: <https://github.com/hplgit/fenics-mixed>. The following command downloads all the files:

```
Terminal> git clone https://github.com/hplgit/fenics-mixed.git
```

The source code for the examples are located in the subdirectory [doc/src/src-fenics-mixed](#). The code examples are tested with FEniCS version 1.2.

2 FEniCS solver with boundary conditions in Fortran

Fortran programs are usually easy to interface in Python by using the wrapper code generator [F2PY](#). F2PY supports Fortran 77, Fortran 90, and even C (and thereby C++, see Section 5). It is our experience that F2PY is much more straightforward to use than the other tools we describe for interfacing Python with compiled languages. F2PY is therefore a natural starting point for our examples.

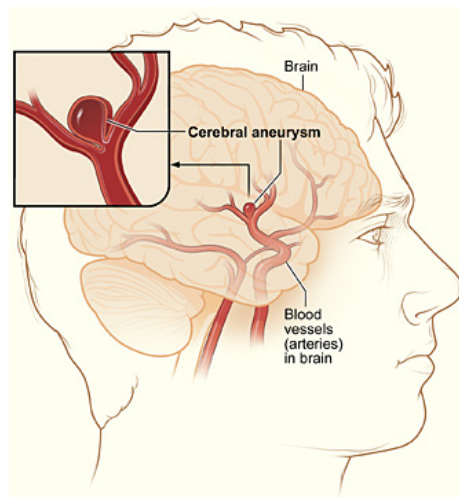


Figure 1: An illustration of a cerebral aneurysm.

The present worked example involves solving the Navier-Stokes equations by a FEniCS solver, but calling up a Fortran 77 code for modeling the boundary conditions. The physical problem concerns blood flow in a cerebral aneurysm. An aneurysm is a balloon-shaped deformation of a cerebral artery, see Figure 1. Some aneurysms rupture and cause stroke, while other remain stable for long periods of time, and it is currently not possible to determine the rupture risk in a patient-specific manner. Computational studies have recently demonstrated that fluid

dynamics simulations can be used to discriminate ruptured from non-ruptured aneurysms [2, 1, 14, 12], retrospectively, and have therefore demonstrated the potential of simulations to many clinicians.

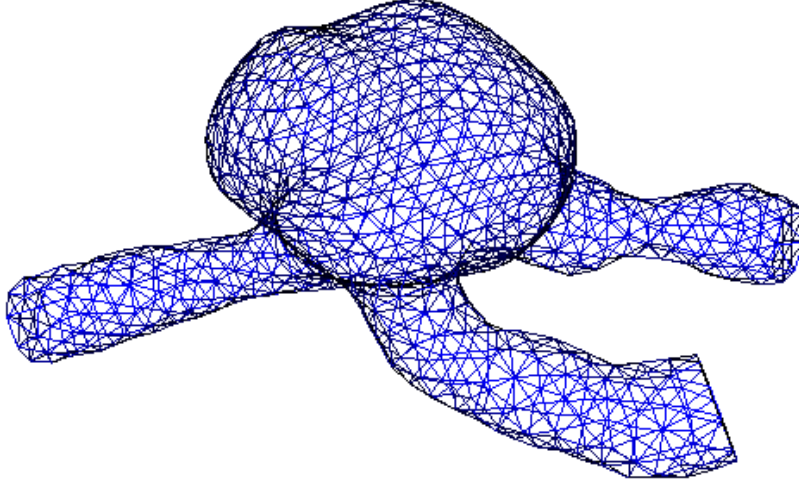


Figure 2: A DOLFIN mesh illustrating a patient-specific aneurysm geometry.

To model blood flow we assume that blood is Newtonian and incompressible with a viscosity of 0.0035 Pa s and density similar to water. The equations read

$$\begin{aligned} \rho \left(\frac{\partial v}{\partial t} + (v \cdot \nabla)v \right) &= -\nabla p + \mu \Delta v + f && \text{in } \Omega && (1) \\ \nabla \cdot v &= 0 && \text{in } \Omega && (2) \end{aligned}$$

Here, v and p are the unknown blood velocity and pressure, respectively, while μ is the viscosity and ρ the density.

Quite often the outlet boundary conditions are unknown. It is therefore common to model the boundary conditions using differential equations of lower dimension. In our case, we assume that the pressure at the inlet or outlet boundaries can be modeled by a system of ODEs:

$$\frac{\partial P_i}{\partial t} = f(P_i, v, p, \dots) \quad \text{on } \partial\Omega_o. \quad (3)$$

These ODEs are coupled to the Navier-Stokes equations through the inlet or outlet boundary condition

$$\mu \frac{\partial v}{\partial n} + pn = P_o \quad \text{on } \partial\Omega_o \quad (4)$$

2.1 The FEniCS solver

The Navier-Stokes solver is implemented in FEniCS as a class `NSSolver`. The typical usage of the class goes as follows:

```
solver = NSSolver()
solver.setIC()
t = 0
dt = 0.01
T = 1.0
P1, P2 = 0, 0
while t < T:
    t += dt
    solver.advance_one_time_step((P1, P2), t)
```

The `setIC()` function sets the initial conditions. Furthermore, `P1` and `P2` are the pressures at the two outlets at time `t`. The implementation details of class `NSSolver` are not essential to this document, so we just refer the reader to the relatively short `NSSolver.py` file.

2.2 The Fortran code for modeling boundary conditions

The `NSSolver` class sets Dirichlet condition for the pressure on inlet and outlet boundaries in terms of prescribed constants in a list `P` (one for each prescribed outlet or inlet). Our aim now is to use a lower-dimensional flow model for computing the Dirichlet values in `P` based on physics and the current velocity and pressure fields. One such model is formulated in terms of ODEs. For an outlet boundary, let P be the pressure at the boundary. Then the model for P is

$$C \frac{\partial P}{\partial t} = Q - P/R_d,$$

where

$$Q = \int_{\partial\Omega_o} v \cdot nds$$

is the volume flux through the boundary $\partial\Omega_o$ (easily computed in the FEniCS solver). The parameters C and R_d must be prescribed along with the initial value of P .

The differential equation for P can be discretized by a very simple Forward Euler scheme. With i denoting the time level corresponding to small time steps δt in the fluid solver time step Δt , we can write

$$P^{i+1} = P^i + \delta t(Q - P/R_d)/C,$$

for $i = 0, \dots, N-1$, where $\Delta t = N\delta t$, and then the new pressure outlet condition is $P = P^N$ for the next time step. P^0 is taken as P at time t (P is the outlet pressure value at time $t + \Delta t$).

The computational model for P is implemented in Fortran. (Our specific model is a simple one; the problem setting is that another research group is continuously developing such models, and their software is in Fortran.) The solver in Fortran is implemented in a file `PMODEL.f` with the content

```

SUBROUTINE PMODEL(P, P_1, R_D, Q, C, N, T)
C   Integrate P in N steps from 0 to T, given start value P_1
INTEGER N
REAL*8 P(0:N), P_1, R_D, Q, C, T
REAL*8 DT
INTEGER I
Cf2py intent(in) P0, R, Q, C, N
Cf2py intent(out) P

    DT = T/N
    P(0) = P_1
    DO I = 0, N-1
        P(i+1) = P(i) + DT*(Q - P(i)/R_D)/C
    END DO
END SUBROUTINE PMODEL

```

Given P_1 as the value of P at time t , the subroutine computes P at all the N local time steps (of length DT) up to time $t+T$, with $P(N)$ as the final value at that time. We shall call `PMODEL` at every time step in the flow solver and let T correspond to the fluid solver time step Δt .

The subroutine is plain Fortran 77 except for some special comment lines starting with `CF2PY`. These are needed because in Fortran, subroutine arguments are both input and output, but in Python one normally takes all input as arguments to a function and *returns* all output arguments. This is technically not possible in Fortran (or C or C++). With the `CF2PY` comment lines we can help the F2PY translator to make the Fortran subroutine look more "Pythonic" from the Python side. To this end, we need to specify what arguments that are input and output. All arguments are input by default, but here we still list them to have complete specification of every argument in this function. The output argument, to be returned to Python, *must* be specified, here P .

Creating a shared library of the Fortran code that we can call from Python as an ordinary module is easy:

```
Terminal> F2PY -c -m bcmodelf77 ../PMODEL.f
```

Here, `-m bcmodelf77` tells F2PY that the module name is be `bcmodelf77`, the `-c` instructs F2PY to compile and create a shared library `bcmodelf77.so`, and `PMODEL.f` is the name of the Fortran file to analyze and compile. Our convention is to compile F2PY modules in a subdirectory of the Fortran code, which explains why the file here has name `../PMODEL.f`.

A little `test code` can compare the Fortran ODE solver with a couple of manual lines in Python:

```

import nose.tools as nt

def test_bcmodelf77():
    import bcmodelf77
    C = 0.127
    R_d = 5.43
    N = 2
    P_1 = 16000
    Q = 1000

```

```

T = 0.01
P_ = bcmodelf77.pmodel(P_1, R_d, Q, C, N, T)

# Manual formula:
P1_ = P_1 + T/2*(Q - P_1/R_d)/C
P1_ = P1_ + T/2*(Q - P1_/R_d)/C
nt.assert_almost_equal(
    P_[-1], P1_, places=10, msg='F77: %g, manual coding: %s'
    %
    (P_[-1], P1_))

if __name__ == '__main__':
    test_bcmodelf77()

```

Note that F2PY turns all upper case letters into lower case when viewed from Python. Also note that this test function is created as a nose unit test. Running `nosetests` in that directory finds all `test_*` functions in all files and executes these functions.

Instead of calling the Fortran function directly with many parameters, we wrap a class around the function such that the syntax of each call to compute P becomes simpler. The idea is to let parameters that are constant through the fluid flow simulation be attributes in the class so that it is sufficient to provide the varying parameters in the call to `PMODEL`. The [Python code](#) hopefully explains this idea clearly:

```

class BCModel:
    def __init__(self, C, R_d, N, T):
        self.C, self.R, self.N, self.T = C, R_d, N, T

    def __call__(self, P, Q):
        P_ = bcmodelf77.pmodel(
            P, self.R, Q, self.C, self.N, self.T)
        return P_

```

We can now set all constant parameters at once,

```

pmodel = BCModel(C=0.127, R_d=5.43, N=2, T=0.01)

```

and there after call the Fortran subroutine `PMODEL` by `pmodel(P, Q)`, i.e., with only the arguments that change from time step to time step in the fluid solver.

2.3 Coupling the Python FEniCS solver with the Fortran routine

It remains to make the final glue between the FEniCS solver and the Fortran subroutine. In the FEniCS solver, we import the `BCModel` class and make a list of such objects, with one element for each outlet boundary where we want to use the ODE model. Then we invoke a time loop where new u and p are computed, then we compute the flux Q , and finally we compute new outlet pressures by calling up each ODE solver in turn. All this code is collected in the file [CoupledSolver.py](#):

```

import NSSolver
from BCModel import BCModel

```

```

solver = NSSolver.NSSolver()
solver.setIC()
t = 0
dt = 0.01 # fluid solver time step
T = 5.0   # end time of simulation

C = 0.127
R = 5.43
N = 1000 # use N steps in the ODE solver in [t,t+dt]

num_outlets = 2 # no out outflow boundaries

# Create an ODE model for the pressure at each outlet boundary
pmodels = [BCModel(C, R, N, dt) for i in range(0,num_outlets)]

P_ = [16000, 100] # start values for outlet pressures

while t < T:
    t += dt

    # Compute u_ and p_ using known outlet pressures P_
    solver.advance_one_time_step(P_, t)
    # Compute the flux at outlet boundaries
    Q = solver.flux()

    # Advance outlet pressure boundary condition to the
    # next time step (for each outlet boundary)
    # (pmodels returns a vector of size N containg the
    # the solution between [t, t+dt].
    # We take the last one with [-1])
    for i in range(0, num_outlets):
        P_[i] = pmodels[i](P_[i], Q[i])[-1]

```

3 FEniCS solver with optimization in Octave

While Python has gained significant momentum in scientific computing in recent years, Matlab and its open source counterpart Octave are still much more dominating tools in the community. There are tons of MATLAB/Octave code around that FEniCS users may like to take advantage of. Fortunately, MATLAB and Octave both have Python interfaces so it is straightforward to call MATLAB/Octave from FEniCS simulators implemented in Python. The technical details of such coupling is the presented with the aid of an example.

3.1 Basic use of Pytave

First we show how to operate Octave from Python. Our focus is on the Octave interface named [Pytave](#). The basic Pytave command is

```
result = pytave.feval(n, "func", A, B, ...)
```

for running the function `func` in the Octave engine with arguments `A`, `B`, and so forth, resulting in `n` returned objects to Python. For example, computing the eigenvalues of a matrix is done by

```
import numpy, pytave
```



```
A = numpy.random.random([2,2])
e = pytave.feval(1, "eig", A)
print 'Eigenvalues:', e
```

The eigenvalues *and* eigenvectors of a generalized eigenvalue problem $Av = \lambda Bv$ is computed by obtain both the eigenvalues and the eigenvectors, we do

```
A = numpy.random.random([2,2])
B = numpy.random.random([2,2])
e, v = pytave.feval(2, "eig", A, B)
print 'Eigenvalues:', e
print 'Eigenvectors:', v
```

Note that we here have two input arguments, A and B, and two output arguments, e and v (and because of the latter the first argument to `feval` is 2).

We could equally well solved these eigenvalue problems directly in `numpy` without any need for Octave, but the next example shows how to take advantage of a package with many MATLAB/Octave files offering functionality that is not available in Python.

3.2 Calling the MATLAB/Octave software

The following scientific application involves the coupling of our FEniCS flow solver with a MATLAB/Octave toolbox for solving optimization problems based on Kriging and the surrogate management method. Our task is to minimize the fluid velocity in a certain region by placing a porous media within the domain. We can choose the size, placement and permeability of the porous media, but are not allowed to affect the pressure drop from the inlet to the outlet much. Figures 3 and 4 show the flow velocity with two different placements of two different porous media.

For this particular application we assume that the Reynolds number is low such that the flow can be modeled by the Stokes problem. Furthermore, the additional resistance caused by the porous medium is modeled by a positive lower order term Ku resulting in the Brinkman model. The equations then reads

$$-\Delta u + Ku - \nabla p = 0, \quad \text{in } \Omega \quad (5)$$

$$\nabla \cdot u = 0, \quad \text{in } \Omega \quad (6)$$

$$u = (0, 0), \quad \text{on } y = 0, 1 \quad (7)$$

$$u = (y(1 - y), 0), \quad \text{on } x = 0 \quad (8)$$

$$\frac{\partial u}{\partial n} + pn = 0, \quad \text{on } x = 1 \quad (9)$$

with

$$K = K_0 \text{ if } |x - x_0| \leq c, \quad |y - 0.5| \leq c,$$

while $K = 0$ outside this rectangular region.

When $K = 0$ we have viscous Stokes flow while inside the porous medium, $K = K_0$, and the Ku term in the equation dominates over the viscous term Δu .

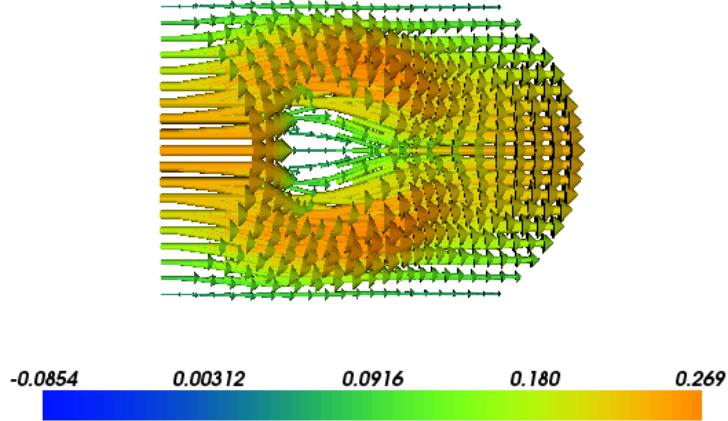


Figure 3: Velocity problem around a porous media with $K_0 = 1000, x_0 = 0.4, c = 0.1$.

The goal functional that we seek to minimize is

$$J(K_0, x_0, c) = u_x|_{(x=1, y=0.5)} + \int_{\Omega} (\nabla p)^2 dx \quad (10)$$

Here, u and p are functions of K_0 , x_0 , and c , and u_x is the x component of u .

The MATLAB/Octave code for the surrogate management and Kriging is based on [Dace](#), but has been extended by Alison Marsden et. al. [10, 9, 8] to implement surrogate management. This algorithm consists of four main steps 1) search, 2) poll, 3) refine and 4) run simulations, with a flow chart appearing in Figure 5. The two first steps find new sample points K_0 , x_0 , and c , while refine increases the resolution in the parameter space, and finally the fourth step runs simulations with the new sample points.

The main algorithm is implemented in Python (file [opimize_flow_by_SMF.py](#)) and listed below. It calls three key Python functions: `search`, `poll`, and `refine`, which make calls to the MATLAB/Octave package.

```
# main loop
while nit <= max_nit and refine_ok and not converged:
    # search step
    if cost_improve:
        Ai_new = search(Aall, Jall, curr_bestA, theta,
                       upb, lob, N, amin, amax, spc, delta)
        prev_it = "search"
        Ai_new = coarsen(Ai_new)
    else:
        # poll step
        if prev_it == "search":
```

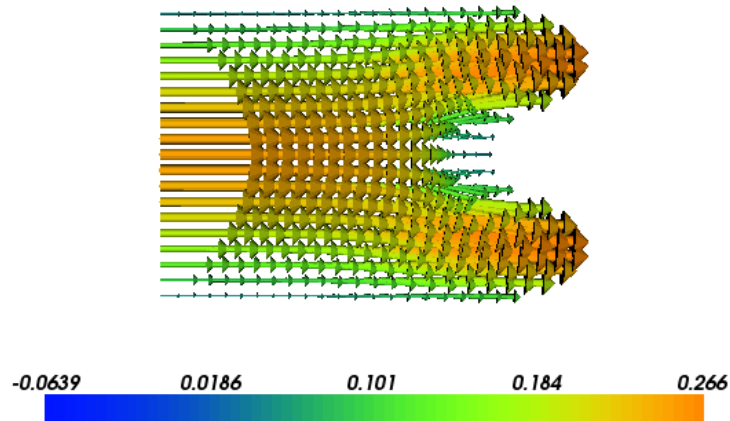


Figure 4: Velocity problem around a porous media with near optimal values: $K_0 = 564, x_0 = 0.92, c = 0.10$.

```

Ai_new = poll(Aall, Jall, curr_bestA, N,
             delta, spc, amin, amax)
prev_it = "poll"
# refine if previous poll did not lead to cost improvement
if prev_it == "poll":
    refine_ok, delta, spc = refine(delta, deltamin, spc)
    if refine_ok:
        Ai_new = search(Aall, Jall, curr_bestA, theta,
                       upb, lob, N, amin, amax, spc,
                       delta)
        prev_it = "search"
    else:
        Ai_new = None
nit += 1

# run simulations on the new parameters
if not Ai_new == None:
    Ai_new, J_new = run_simulations(Ai_new)

# stack the new runs to the previous
Jall = numpy.hstack((Jall, J_new))
Aall = numpy.vstack((Aall, Ai_new))

# monitor convergence (write to files)
monitor(Aall, Jall, nit, curr_bestA, curr_bestJ,
        delta, prev_it, improve, spc)

# check convergence
cost_improve, curr_bestA, curr_bestJ = check(
    Ai_new, J_new, nit, curr_bestJ, curr_bestA)

```

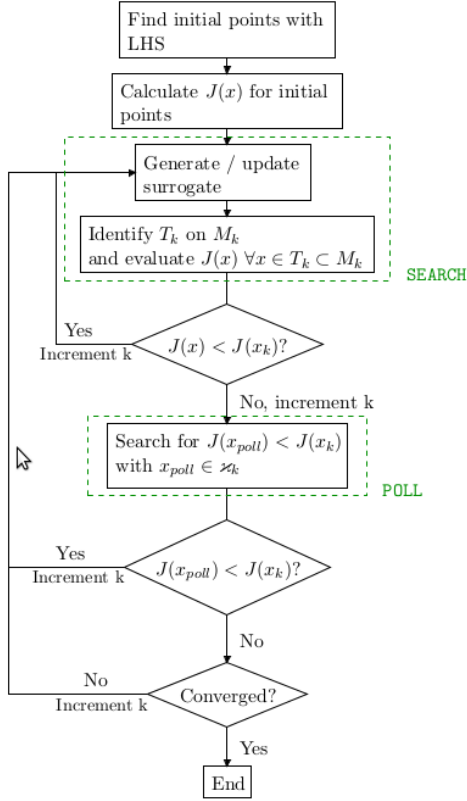


Figure 5: The flow chart of the surrogate management method.

```

else:
    cost_improve = 0
  
```

The search and poll steps are both implemented in Python but are mainly wrappers around MATLAB/Octave functions. The search step is implemented as follows:

```

def search(Aall, Jall, curr_bestA, theta, upb,
          lob, N, amin, amax, spc, delta):
    """Search step."""
    # make sure that all points are unique
    (Am, Jm) = pytave.feval(2, "dsmerge", Aall, Jall)

    next_ptsall = []
    next_pts = None
    max_no_searches = 100
    no_searches = 0
    while next_pts == None and no_searches < max_no_searches:
        next_ptsall, min_est, max_mse_pt = pytave.feval(
  
```

```

3, "krig_min_find_MADS_oct",
Am, Jm, curr_bestA, theta, upb, lob,
N, amin, amax, spc, delta)
next_pts = check_for_new_points(next_ptsall, Aall)
no_searches += 1
return next_pts

```

Here, `dsmerge` and `krig_min_find_MADS_oct` are functions available in the MATLAB/Octave files `dsmerge.m` and `krig_min_find_MADS_oct.m`. We need to notify Octave about the directory (SMF) where these files can be found:

```
pytave.feval(0, "addpath", "SMF")
```

3.3 The FEniCS PDE solver

The [FEniCS solver](#) first defines the inflow condition (class `EssentialBC`), the K coefficient in the PDEs, and the Dirichlet boundary:

```

class EssentialBC(Expression):
    def eval(self, v, x):
        if x[0] < DOLFIN_EPS:
            y = x[1]
            v[0] = y*(1-y); v[1] = 0
        else:
            v[0] = 0; v[1] = 0

    def value_shape(self):
        return (2,)

class K(Expression):
    def __init__(self, K0, x0, c):
        self.K0, self.x0, self.c = K0, x0, c

    def eval(self, v, x):
        x0, K0, c = self.x0, self.K0, self.c
        if abs(x[0] - x0) <= c and abs(x[1] - 0.5) <= c:
            v[0] = K0
        else:
            v[0] = 1

def dirichlet_boundary(x):
    return bool(x[0] < DOLFIN_EPS or x[1] < DOLFIN_EPS or \
               x[1] > 1.0 - DOLFIN_EPS)

```

The core of the solver is the following class:

```

class FlowProblem2Optimize:
    def __init__(self, K0, x0, c, plot):
        self.K0, self.x0, self.c, self.plot = K0, x0, c, plot

    def run(self):
        K0, x0, c = self.K0, self.x0, self.c

        mesh = UnitSquareMesh(20, 20)
        V = VectorFunctionSpace(mesh, "Lagrange", 2)
        Q = FunctionSpace(mesh, "Lagrange", 1)
        W = MixedFunctionSpace([V,Q])
        u, p = TrialFunctions(W)

```

```

v, q = TestFunctions(W)
k = K(K0, x0, c)

u_inflow = EssentialBC()
bc = DirichletBC(W.sub(0), u_inflow, dirichlet_boundary)
f = Constant(0)

a = inner(grad(u), grad(v))*dx + k*inner(u, v)*dx + \
    div(u)*q*dx + div(v)*p*dx
L = f*q*dx

w = Function(W)
solve(a == L, w, bc)

u, p = w.split()
u1, u2 = split(u)

goal1 = assemble(inner(grad(p), grad(p))*dx)
goal2 = u(1.0, 0.5)[0]*1000
goal = goal1 + goal2

if self.plot:
    plot(u)

key_variables = dict(K0=K0, x0=x0, c=c, goal1=goal1,
                    goal2=goal2, goal=goal)
print key_variables
return goal1, goal2

```

3.4 Coupling FEniCS and the MATLAB/Octave software

It now remains to do the coupling of the optimization algorithm that makes use of MATLAB/Octave files and the FEniCS flow solver. The following function performs the task:

```

def run_simulations(Ai):
    """Run a sequence of simulations with input parameters Ai."""
    import flow_problem
    plot = True
    if len(Ai.shape) == 1: # only one set of parameters
        J = numpy.zeros(1)
        K0, x0, c = Ai
        p = flow_problem.FlowProblem2Optimize(K0, x0, c, plot)
        goal1, goal2 = p.run()
        J[0] = goal1 + goal2
    else: # several sets of parameters
        J = numpy.zeros(len(Ai))
        i = 0
    for a in Ai:
        K0, x0, c = a
        p = flow_problem.FlowProblem2Optimize(K0, x0, c, plot)
        goal1, goal2 = p.run()
        J[i] = goal1 + goal2
        i = i+1
    return Ai, J

```

3.5 Installing Pytave

Obviously, Pytave depends on Octave, which can be somewhat challenging to install. Prebuilt binaries are available for Linux (Debian/Ubuntu, Fedora, Gentoo, SuSE, and FreeBSD), Mac OS X (via MacPorts or Homebrew), and Windows (requires Cygwin). On Debian-based systems (including Ubuntu) you are recommended to run these commands

```
# Install Octave
sudo apt-get update
sudo apt-get install libtool automake libboost-python-dev libopenmpi-dev
sudo apt-get install octave octave3.2-headers

# Install Pytave
bzip2 -d pytave.tar.gz
cd pytave
bzip2 -d pytave-51.tar.gz
autoreconf --install
./configure
sudo python setup.py install
```

Pytave has not yet been officially released, but it is quite stable and has a rather complete interface to Octave. Unfortunately, the latest changeset has a bug and that is why we need to revert to a previous revision (`bzip2 -d pytave-51.tar.gz`).

There are at least two Python modules that interface MATLAB: [pymat2](#) and [pymatlab](#), but the authors do not have MATLAB installed and were unable to test these packages.

4 How to interface a C++/DOLFIN code from Python

Although FEniCS can easily and flexibly be extended by Python code, the need for speed in scientific computing occasionally makes a demand to implement new finite element functionality in C++. The present example shows how one can extend DOLFIN's finite element functionality through a new piece of C++ code and call this functionality from a Python FEniCS solver.

FEniCS finite element functions can be evaluated at an arbitrary point in the mesh. In a parallel computing setting, however, the evaluation point must be in the part of the mesh that belongs to the current process and the searching for the element containing the point is not optimally efficient. Therefore, one may want to have a utility for fast evaluation of finite element functions at prescribed points on parallel computers. We can write a class `Probe` for this purpose. The constructor takes a spatial point `x` and precomputes which element that contains the point and other data useful for later fast evaluation of functions at `x`. A member function `eval(u)` takes any `Function` object `u` and stores its value(s) at the point `x`. With `get_values(i)` we can retrieve all values component `i` of the function computed in previous calls to `eval`. For a scalar function there is only one component (`i=0`), but the class supports vector and tensor functions too.

The name `Probe` reflects the use of such a class: we insert a probe, as in a physical experiment, and measure the response at that point through time. In FEniCS simulators it means that we want to record the evolution in time of

some field at a given spatial point. For long time series there can be a lot of evaluations of the field at this point, and class `Probe` will be much more efficient than the standard FEniCS point evaluation of fields (which performs a lot of searching to find the element containing the point).

4.1 The C++ class

Class `Probe` is a fairly short C++ code that makes use of various DOLFIN C++ classes and programming conventions. The header file reads

```
#include <dolfin/function/FunctionSpace.h>
#include <dolfin/function/Function.h>

namespace dolfin
{
  class Probe
  {
  public:
    Probe(const Array<double>& point, const FunctionSpace& V);
    void eval(const Function& u);
    std::vector<double> get_values(std::size_t component);

    std::size_t num_components() {return value_size_loc;};
    std::size_t number_of_eval_calls() {return
      _probes[0].size();};
    std::vector<double> get_point();
    void erase(std::size_t i);
    void clear();

  private:
    std::vector<std::vector<double> > basis_matrix;
    std::vector<double> coefficients;
    double _x[3];
    boost::shared_ptr<const FiniteElement> _element;
    Cell* dolfin_cell;
    UFCell* ufc_cell;
    std::size_t value_size_loc;
    std::vector<std::vector<double> > _probes;
  };
}
```

The most important functionality for users lies in the constructor and the `eval` and `get_values` functions, while the rest of the class contains short convenience functions and data structures for help with fast function evaluations. Note that `eval` does not return any value, it just records the value.

The reader may consult the corresponding `Probe.cpp` file for all implementation details. Obviously, this type of code requires familiarity with the DOLFIN classes, but looking at the DOLFIN code itself is a good starting point for learning about those classes, the associated implementation conventions, and other programming tools that the DOLFIN library makes use of.

4.2 Compiling and linking at the Python DOLFIN level

The next, and often more technically challenging, step is to compile the C++ code, link it to DOLFIN, and make it callable from a FEniCS solver in Python.

Fortunately, this is not so difficult if we use the FEniCS Just-in-time (JIT) compiler Instant, which is already the compiler that DOLFIN applies when compiling variational forms. Instant employs SWIG in its JIT compiling, and some knowledge of SWIG is therefore required to understand how Instant works. However, hardly any SWIG knowledge is needed if we use the convenience function `compile_extension_module` found in the Python package `dolfin` package. This function is a high-level interface to Instant functionality.

Basically, the `compile_extension_module` function requires a declaration of our C++ code to be interfaced, a list of `.cpp` source code files, and some information on where files are found. Compilation and linking are then taken care of automatically. The C++ code to be interfaced in this example is contained in the `Probe.h` header file. The call to the `compile_extension_module` function is then

```
from dolfin import *
import numpy
import os

header_file = open("Probe/Probe.h", "r")
code = header_file.read()
header_file.close()
probe_module = compile_extension_module(
    code=code, source_directory="Probe", sources=["Probe.cpp"],
    include_dirs=[".", os.path.abspath("Probe")])
```

We can now import `probe_module` in the forthcoming code and use it for fast evaluations at some point `x`:

```
mesh = UnitCubeMesh(10, 10, 10)
V = FunctionSpace(mesh, 'CG', 1)

x = numpy.array((0.5, 0.5, 0.5))
probe = probe_module.Probe(x, V)

u0 = interpolate(Expression('x[0]'), V)
# Fast evaluation of U0 at x:
probe.eval(u0)
print "The number of probes is ", probe.number_of_eval_calls()
print "The value at ", x, " is ", probe.get_values(0)
```

To summarize, with `compile_extension_module` the compilation and linking of C++ and DOLFIN code to make accessible in Python is a matter of one function call.

4.3 Compiling and linking at the Instant level

We shall now go into details how the steps above would be done by using basic Instant only, as this explains how to use Instant for interfacing C++ code in general. Instant provides the `build_module` function for building a Python module out of the C++ code:

```
compiled_module = instant.build_module(
    code=code,
    source_directory=source_dir,
    additional_declarations=additional_decl,
```

```

system_headers=system_headers,
include_dirs=include_dirs,
swigargs=swigargs,
sources=sources,
cmake_packages=cmake_packages)

```

Here,

- `code` is the C++ code that is to be wrapped,
- `source_directory` is the directory where the C++ `.cpp` files are found,
- `additional_declarations` are additional declaration needed to make SWIG behave properly,
- `system_headers` is a list of the additional header files needed for compilation,
- `include_dirs` is a list of additional include directories required for compilation,
- `swigargs` is the arguments that shall be passed to SWIG on the command line,
- `sources` is a list of C++ files that shall be compiled into the Python module, and
- `cmake_packages` is a list of packages that the CMake compilation depend on.

The following code illustrates the setting of these variables:

```

system_headers = ['numpy/arrayobject.h',
                  'dolphin/function/Function.h',
                  'dolphin/function/FunctionSpace.h']
swigargs = ['-c++', '-fcompact', '-O', '-I.', '-small']
cmake_packages = ['DOLFIN']
sources = ["Probe.cpp"]
source_dir = "Probe"

```

The `Probe` class employs several DOLFIN classes. Hence, for this class to work properly it is crucial that the JIT compiler and SWIG are told how to relate to the DOLFIN classes. Instant provides the hook `additional_declarations` for providing additional declarations to SWIG. Such declarations require knowledge of how to write SWIG interface files. In the current example, the additional declarations look like the following string:

```

additional_decl = """
%init%{
import_array();
%}

// Include global SWIG interface files:
// Typemaps, shared_ptr declarations, exceptions, version

```

```

#include <boost_shared_ptr.i>

// Global typemaps and forward declarations
#include "dolfin/swig/typemaps/includes.i"
#include "dolfin/swig/forwarddeclarations.i"

// Global exceptions
#include <exception.i>

// Local shared_ptr declarations
%shared_ptr(dolfin::Function)
%shared_ptr(dolfin::FunctionSpace)

// %import types from submodule function of SWIG module function
%import(module="dolfin.cpp.function")
    "dolfin/function/Function.h"
%import(module="dolfin.cpp.function")
    "dolfin/function/FunctionSpace.h"

%feature("autodoc", "1");
"""

```

The `init` part containing `import_array` is always needed when NumPy [11] arrays are involved. Thereafter, we include various SWIG interface files (ending in `.i`) that we need in DOLFIN-related code. We also need shared pointers for `dolfin::Function` and `dolfin::FunctionSpace`. In addition we need header files for `Function` and `FunctionSpace` classes in DOLFIN. Note that the `import` and `include` statements in SWIG may seem similar, but that whereas `include` makes SWIG generate wrappers for the code included, the `import` directive simply provides SWIG with the necessary type information.

We refer to the complete file [instant_test_probe.py](#) for how all of the information about is put together and executed in order to build the extension module using plain Instant functionality.

5 FEniCS solver coupled with ODE solver in C++

In this final example we will consider a solver for a reaction-diffusion equation described by a parabolic PDE coupled to a set of ODEs. The equation can be written as

$$\begin{aligned}
 u_t &= \Delta u + f(u, s), & \forall x \in \Omega, t > 0, \\
 s_t &= g(u, s), & \forall x \in \Omega, t > 0.
 \end{aligned}$$

Here, u is a scalar function, subscript t means differentiation with respect to time, s is a scalar field (governed pointwise by an ODE), and Δu is a Laplace term. The problem is usually solved using a first order operator splitting scheme, where we at a time level first solve the PDE with f evaluated at the previous time level, and thereafter we update the ODEs using the most recent value of u . More precisely,

$$u^n = u^{n-1} + \Delta t(\Delta u^n + f(u^{n-1}, s^{n-1})),$$

$$s^n = s^{n-1} + \Delta t g(u^n, s^{n-1}),$$

The superscript n denotes the time level, and Δt is the time step.

The solver for the parabolic problem is implemented in FEniCS, while the ODE solver is implemented in a homemade C++ code. We will glue these two different solvers together using Python. The C++ code consists basically of the class `ODEFieldSolver` declared in a file [ODEFieldSolver.h](#):

```
class ODEFieldSolver {
    int n;           // no of points (or regions)
    double* s;      // discrete values of unknown s
    double* s_1;    // s at the previous time level
    double* u;      // discrete values of external field u
    double dt;      // time step size

public:
    ODEFieldSolver();
    ~ODEFieldSolver();
    void redim(int n); // allocate data structures
    int size();       // return the no of points/regions
    virtual double g(double s, double u);
    void set_dt(double dt);
    void set_IC(int n, double* in_array);
    void set_u (int n, double* in_array);
    void set_IC(double const_value);
    void set_u (double const_value);
    void advance_one_timestep();
};
```

The `set_IC` functions set the initial condition of the ODE system, and `set_u` provides the u field (the "environment") to the ODE system. Note that there are two versions of `set_IC` and `set_u`: one for a constant value one for spatial variations. We refer to the [ODEFieldSolver.cpp](#) file for implementational details.

The mathematics behind the shown class is to have n regional or pointwise values of u and s , and then solve $s_t = g(s, u)$ in each region or at each point. In the present case we will solve for s at the nodes in the finite element mesh used to compute u . The s and u functions are in the C++ code represented by plain C arrays holding nodal values. The usage of the C++ code typically goes like this:

```
ODEFieldSolver solver = ODEFieldSolver();
solver.redim(n);           // allocate data
solver.set_dt(dt);        // set time step
solver.set_IC(n, s0);     // set initial conditions
t = 0
while (t <= T) {
    solver.set_u(n, u);    // give access to PDE solution
    solver.advance_one_timestep();
    // plot solver.s
}
```

A subclass must be written to specify the desired g function.

We need to wrap the C++ class in Python such that the FEniCS Python solver can call the C++ code. We would then need to transfer the computed s back to Python. To this end, we add a member function `get_s` to the class so that we can fill some array on the user's side with the most recently computed s values:

```
class ODEFieldSolver {
    ...
    void get_s (int& n, double* out_array);
};
```

5.1 Wrapping with F2PY

The easiest way to interface Fortran, C, and C++ code is to use F2PY. Although F2PY was made for interfacing Fortran and most of the documentation is written with Fortran codes in mind, it is convenient to interface C and C++ too. Or more precisely, F2PY can interface a pure C API, not C++ classes. The idea is then to construct a set of C functions on top of the C++ classes for accessing high-level operations using the those classes. The example involving class `ODEFieldSolver` will illustrate the elements in this technique.

C API to C++ code. The first step is to decide on the C API. The exposed functions in Python must do essentially the same as the main program. A possible set of functions is

- `set_ic_and_dt(int n, double* s0, double dt)` for initializing the class object and setting the initial conditions and the time step. Also a variant `set_const_ic_and_dt` for constant initial condition s_0 is handy.
- `set_u(int n, double* u)` for assigning the u function to the class.
- `advance_one_timestep()` for computing the solution at a time step.
- `get_s(int n, double* s)` for getting access to the computed array s in the `ODEFieldSolver` class.

These functions must make use of a global variable holding a `ODEFieldSolver` object and interact with this object as appropriate. The [complete code of the C API](#) then becomes

```
#include "ODEFieldSolver.h"

ODEFieldSolver solver = ODEFieldSolver();

extern "C" {

void set_ic_and_dt(int n, double* s0, double dt)
{
    solver.redim(n);
    solver.set_dt(dt);
    solver.set_IC(n, s0);
}
```

```

void set_const_ic_and_dt(int n, double s0, double dt)
{
    solver.redim(n);
    solver.set_dt(dt);
    solver.set_const_IC(s0);
}

void set_u(int n, double* u)
{
    solver.set_u(n, u);
}

void advance_one_timestep()
{
    solver.advance_one_timestep();
}

void get_s(int n, double* s)
{
    solver.get_s(n, s);
}
}

```

Writing corresponding Fortran signatures. The nice thing about F2PY is that it can automatically make a Python interface to this C code, where NumPy arrays can be passed to the functions taking plain C arrays as arguments. For this to work, F2PY needs a specification of all the C functions in terms of Fortran 90 module syntax. However, F2PY can generate this module for us if we specify the function signatures in plain Fortran 77. This is done as follows:

```

      subroutine set_ic_and_dt(n, s0, dt)
Cf2py intent(c) set_ic_and_dt
      integer n
      real*8 s0(0:n-1), dt
Cf2py intent(c) n, s0, dt
      return
      end

      subroutine set_const_ic_and_dt(n, s0, dt)
Cf2py intent(c) set_const_ic_and_dt
      integer n
      real*8 s0, dt
Cf2py intent(c) n, s0, dt
      return
      end

      subroutine set_u(n, u)
Cf2py intent(c) set_u
      integer n
      real*8 u(0:n-1)
Cf2py intent(c) n, u
      return
      end

```

```

subroutine advance_one_timestep()
Cf2py intent(c) advance_one_timestep
return
end

subroutine get_s(n, s)
Cf2py intent(c) get_s
integer n
Cf2py intent(c) n
real*8 s(0:n-1)
Cf2py intent(c, in, out) s
return
end

```

For each C function we

- write the corresponding Fortran subroutine or function header,
- insert an F2PY-specific comment (CF2PY) that tells that the function is in C: `intent(c)`,
- specify that all variables are in C: `intent(c)` (Fortran treats all arguments as pointers, so the specification of C variables is strictly needed only for non-pointers),
- specify if we want the Python interface to return one or more output arguments.

Regarding the last point, we specify `s` in `get_s` as `intent(c,in,out)`, meaning that we in Python can call this function as `s = get_s(s)`. The `s` argument is needed for the function to avoid reallocating the returned array every time the function is call. Instead we reuse the storage provided in the `s` array.

If the Fortran 77 signatures are in a file `signatures_capi2cpp.f` we can get F2PY to generate a Fortran 90 module in a file `ODEFieldSolvercpp.pyf` by the command

```
Terminal> F2PY -m ODEFieldSolvercpp -h ODEFieldSolvercpp.pyf \
--overwrite-signature signatures_capi2cpp.f
```

The `-m` option specifies the name of the extension module that contains the Python interfaces to the C API.

The module typically looks like

```

!      -*- f90 -*-
! Note: the context of this file is case sensitive.

python module ODEFieldSolvercpp ! in
interface ! in :ODEFieldSolvercpp
subroutine set_ic_and_dt(n,s0,dt)
intent(c) set_ic_and_dt
integer,
optional, intent(c), check(len(s0)>=n), depend(s0) :
: n=len(s0)
real*8 dimension(n), intent(c) :: s0

```

```

    real*8 intent(c) :: dt
end subroutine set_ic_and_dt
subroutine set_const_ic_and_dt(n,s0,dt)
    intent(c) set_const_ic_and_dt
    integer intent(c) :: n
    real*8 intent(c) :: s0
    real*8 intent(c) :: dt
end subroutine set_const_ic_and_dt
subroutine set_u(n,u)
    intent(c) set_u
    integer,
        optional, intent(c), check(len(u)>=n), depend(u) ::
        n=len(u)
    real*8 dimension(n), intent(c) :: u
end subroutine set_u
subroutine advance_one_timestep ! in :
    ODEFieldSolvercpp:signatures_capi2cpp.f
    intent(c) advance_one_timestep
end subroutine advance_one_timestep
subroutine get_s(n,s)
    intent(c) get_s
    integer,
        optional, intent(c), check(len(s)>=n), depend(s) ::
        n=len(s)
    real*8 dimension(n), intent(c,in,out) :: s
end subroutine get_s
end interface
end python module ODEFieldSolvercpp

! This file was auto-generated with f2py (version:2).
! See http://cens.ioc.ee/projects/f2pye/

```

Those who are familiar with Fortran 90 modules can write such code by hand instead of first writing Fortran 77 headers and letting F2PY generate the module.

Building the extension module. With the aid of the Fortran 90 specification of the C functions, F2PY can compile and link the extension module by a command like

```

Terminal> F2PY -c --fcompiler=gfortran -I.. --build-dir tmp1 \
-DF2PY_REPORT_ON_ARRAY_COPY=1 \
ODEFieldSolvercpp.pyf ../ODEFieldSolver.cpp capi2cpp.cpp

```

The `-DF2PY_REPORT_ON_ARRAY_COPY=1` option is handy for letting F2PY notify us if arrays are copied when transferred from Python to C, as we want to avoid time-consuming copies. The C++ class is assumed to be in the parent directory (note `-I..` and the `../` prefix in the filename). All the files generated and built by F2PY will reside in the `tmp1` directory for later inspection if run into build problems.

The result of the above compile command is a C/C++ extension module in the file `ODEFieldSolvercpp.so`. The module can be loaded into Python and examined for content:

```

>>> import ODEFieldSolvercpp
>>> dir(ODEFieldSolvercpp)

```



```
[ '__doc__', '__file__', '__name__', '__package__', '__version__',
'advance_one_timestep', 'get_s', 'set_const_ic_and_dt',
'set_ic_and_dt', 'set_u']
>>> print ODEFieldSolvercpp.__doc__
This module 'ODEFieldSolvercpp' is auto-generated with F2PY (version:2).
Functions:
  set_ic_and_dt(s0,dt,n=len(s0))
  set_const_ic_and_dt(n,s0,dt)
  set_u(u,n=len(u))
  advance_one_timestep()
  s = get_s(s,n=len(s))
```

A word of caution is required for newcomers to F2PY: it is extremely important to *study the doc strings* of the various functions before trying to call them from Python. The reason is that F2PY drops unnecessary arguments, such as array lengths (since these are contained in NumPy array objects), and all output arguments are returned and removed from the subroutine's argument list. The function arguments and return values are therefore different in Python and C! For example, the `set_ic_and_dt` function only needs `s0` transferred from Python since `n` can be deduced from the F2PY-generated interface. The signature of this function, as seen from Python, is then

```
>>> print ODEFieldSolvercpp.set_ic_and_dt.__doc__
set_ic_and_dt - Function signature:
  set_ic_and_dt(s0,dt,[n])
Required arguments:
  s0 : input rank-1 array('d') with bounds (n)
  dt : input float
Optional arguments:
  n := len(s0) input int
```

Furthermore, the `get_s` function has specified its `s` argument as input and output (`intent(c,in,out)`) and the doc string shows the correct call syntax:

```
>>> print ODEFieldSolvercpp.get_s.__doc__
get_s - Function signature:
  s = get_s(s,[n])
Required arguments:
  s : input rank-1 array('d') with bounds (n)
Optional arguments:
  n := len(s) input int
Return objects:
  s : rank-1 array('d') with bounds (n)
```

Main program in Python. The Python code for calling the C++ functions in the `ODEFieldSolvercpp` module can take the following form:

```
import ODEFieldSolvercpp as solver
import numpy

s0 = numpy.array([0, 1, 4], float) # ensure float elements!
u = numpy.array([0, 1, 1], float)
n = s0.size
s = numpy.zeros(n)

solver.set_ic_and_dt(s0, dt=0.1)
for n in range(1, 8):
    solver.set_u(u)
```

```

solver.advance_one_timestep()
s = solver.get_s(s)
print n, s

```

5.2 A pure C version of the C++ class

It may be illustrative to also see a pure C code that implements the same type of actions as the C++ class. The class variables are here global variables in a library and all the class functions are stand-alone C functions working with these global variables. A bit more sophisticated implementation would collect the global variables in a global struct instead, so that the functions work with the struct. The advantage of a pure C code is that F2PY can interface all parts of this code directly without any need to make a C API to C++ code. (Having said that, we should add that making a C API to C++ codes is often a good exercise as it tends to emphasize faster computing with arrays rather than with special (potentially small) C++ objects. Python interfacing of C++ this way may lead to sound redesign of the C++ code.)

The pure C implementation goes as follows:

```

#include <stdlib.h>
#include <stdio.h>

/* global variables */
double* s;
double* s_1;
double* u;
double dt;
int n;

void redim(int n_)
{
    n = n_;
    s = malloc(sizeof(double)*n);
    s_1 = malloc(sizeof(double)*n);
    u = malloc(sizeof(double)*n);
}

void deallocate()
{
    free(s); free(s_1); free(u);
}

/* Note: do not mix upper and lower case letters as in set_IC_...
   This leads to undefined symbols when f2py compiles the code.
*/

void set_ic_and_dt(int n_, double* s0, double dt_)
{
    int i;
    redim(n_);
    dt = dt_;
    for (i=0; i<n; i++) {
        s_1[i] = s0[i];
    }
}

```

```

}

void set_const_ic_and_dt(int n_, double s0, double dt_)
{
    int i;
    redim(n_);
    dt = dt;
    for (i=0; i<n; i++) {
        s_1[i] = s0;
    }
}

void set_u(int n_, double* u_)
{
    int i;
    for (i=0; i<n; i++) {
        u[i] = u_[i];
    }
}

double g(double s_, double u_) {
    /* return s_*u_*(1 - s_); */
    return s_;
}

void advance_one_timestep()
{
    /* Use the Forward Euler time integration for simplicity */
    int i;
    for (i=0; i<n; i++) {
        s[i] = s_1[i] + dt*g(s_1[i], u[i]);
        /* For debugging: */
        /* printf("i=%d, s_1=%g, dt=%g, g=%g, s=%g\n",
            i, s_1[i], dt, g(s_1[i], u[i]), s[i]); */
    }
    /* Update for next time step */
    for (i=0; i<n; i++) { s_1[i] = s[i]; }
}

void get_s(int n_, double* s_)
{
    int i;
    for (i=0; i<n; i++) {
        s_[i] = s[i];
    }
}
}

```

By writing the corresponding Fortran 77 signatures, F2PY can generate a Fortran 90 module specification of the extension module, and this code can be compiled as explained above. We refer to the files in the `f2py-c` directory for details.

5.3 Wrapping with SWIG

Next, we employ the tool `SWIG` to wrap the C++ class directly and make it available as a Python class. `SWIG` is also used in `DOLFIN` and `Instant`, as

demonstrated in Section 4. Useful references on SWIG in a FEniCS context are [6, 7, 13, 3].

To use SWIG, you must first write an *interface file* (ending in `.i`) that tells SWIG about the parts of the C++ code you want to access from Python. The next step is to run SWIG to generate (a lot of) wrapper code in C. The final step is to compile the wrapper code and your C++ code and link with required libraries.

A first attempt to write an interface for our `ODEFieldSolver` class consists in listing just the class declaration:

```
%module ODEFieldSolver
%{
#include <arrayobject.h>
#include <sstream>
#include "ODEFieldSolver.h"
%}

%init %{
import_array();
%}

class ODEFieldSolver {
    int n;           // no of points (or regions)
    double* s;       // discrete values of unknown s
    double* s_1;     // s at the previous time level
    double* u;       // discrete values of external field u
    double dt;       // time step size

public:
    ODEFieldSolver();
    ~ODEFieldSolver();
    void redim(int n); // allocate data structures
    int size();       // return the no of points/regions
    virtual double g(double s, double u);
    void set_dt(double dt);
    void set_IC(int n, double* in_array);
    void set_u (int n, double* in_array);
    void set_IC(double const_value);
    void set_u (double const_value);
    void advance_one_timestep();

    void get_s (int& n, double* out_array);
};
```

All SWIG commands start with `%`. The `%module` command defines the name of the module. Following this command comes a list of header files needed by the module. The `%init` command includes code that should be executed when the module is imported in Python. When using NumPy arrays in C++ code we always need to call the `import_array` function to initialize the NumPy package (removal of this statement will result in a segmentation fault!). The rest of the code defines the interface that should be wrapped, that is; the declaration of the class `ODEFieldSolver`.

SWIG is meant to automate interfacing of C and C++ code, and there is mainly only one thing that needs to be addressed manually: the handling of pointers to arrays. Consider for instance the `set_IC` function. Here, `in_array` is a pointer to the first element of a double precision array of length `n`. However, the fact that `in_array` is an array is not explicitly stated in C++, and therefore SWIG simply by default handles the pointer as a plain pointer, and this is not what we want. SWIG does, however, offer typemaps for changing this default behavior. With typemaps we can specify that the pointer `in_array` is a NumPy array object (`PyObject`) when it comes from Python, and we can extract the underlying data pointer (`double*`) and communicate it to C.

To enable NumPy arrays to be passed to the functions `set_IC` and `set_u` we provide the following typemap.

```
%typemap(in) (int n, double* array){
    if (!PyArray_Check($input)) {
        PyErr_SetString(PyExc_TypeError, "Not a NumPy array");
        return NULL; ;
    }
    PyArrayObject* pyarray;
    pyarray = (PyArrayObject*)$input;
    if (!(PyArray_TYPE(pyarray) == NPY_DOUBLE)) {
        PyErr_SetString(PyExc_TypeError, "Not a NumPy array of
            doubles");
        return NULL; ;
    }
    $1 = int(pyarray->dimensions[0]);
    $2 = (double*)pyarray->data;
}
```

Typemap code often looks complicated, at least when viewed for the first time. The logic is straightforward, though, once some basic knowledge of the C API of Python and NumPy is acquired. The idea with the typemap is to recognize a set of arguments in C/C++, here `n` and `in_array`, and then execute some C/C++ code to transform a Python object to the C/C++ arguments. In the present example we want to map a NumPy array object to an integer `n` (the array size) and a plain C array `in_array` (the array data). All Python objects, when viewed in C, are of type `PyObject`. We can think of `PyObject` as a superclass for all the different object types in Python. The special NumPy array object type is `PyArrayObject`.

SWIG has some special variables prefixed with `$`, which in the present example are `$input` for the incoming NumPy array object, and `$1` and `$2` for the outgoing C/C++ arguments `n` and `in_array`. The first `if` statement checks that the incoming array is of right type, and if not, a `TypeError` exception is raised. The `return NULL` statement is essential for this exception to work. The next step is to cast the `PyObject` pointer in `$input` to the correct array object type, `PyArrayObject`, because we need this object to call C functionality in the NumPy object to extract the data and the array size. For safety reasons, we insert a test that the array data are of type `NPY_DOUBLE` so that the array element types in Python and C match. Then we come to the final and most

essential point: extracting data from the NumPy array object and storing them in `n` (\$1) and `in_array` (\$2):

```
$1 = int(pyarray->dimensions[0]);
$2 = (double*)pyarray->data;
```

Because we have overloaded the `set_IC` function, we also need to provide SWIG with a `typecheck` to determine which of the C++ functions to use. A suitable `typecheck` is:

```
%typecheck(SWIG_TYPECHECK_DOUBLE_ARRAY) (int n, double* array) {
$1 = PyArray_Check($input) ? 1 : 0;
}
```

The function

```
void get_s(int n, double *array);
```

should return NumPy arrays when called from Python as

```
s = odesolver.get_s()
```

That is, we would like to be able to call this function from Python without providing an input array, and instead get an output array. This means that an array must be created before being passed to C++ and then returned to Python. To accomplish this we hide the function by calling it `_get_s`. Then we extend the interface using the `%extend` and `%pythoncode` directives with a Python function `get_s`. The Python function `get_s` allocates an array before passing it to the hidden `_get_s` function and thereafter it returns the array. The code is

```
/* Wrap ODEFieldSolver::get_s in a Python function */
%rename (_get_s) ODEFieldSolver::get_s;

%extend ODEFieldSolver{
%pythoncode%{
    def get_s(self):
        import numpy as np
        a = np.zeros(self.size())
        self._get_s(a)
        return a
%}
}
```

To summarize, the complete SWIG interface file for wrapping the `ODEFieldSolver` class is listed next.

```
%module ODEFieldSolver
%{
#include <arrayobject.h>
#include <sstream>
#include "ODEFieldSolver.h"
%}

%init %{
```

```

import_array();
%}

%typecheck(SWIG_TYPECHECK_DOUBLE_ARRAY) (int n, double* array) {
$1 = PyArray_Check($input) ? 1 : 0;
}

%typemap(in) (int n, double* array){
  if (!PyArray_Check($input)) {
    PyErr_SetString(PyExc_TypeError, "Not a NumPy array");
    return NULL; ;
  }
  PyArrayObject* pyarray;
  pyarray = (PyArrayObject*)$input;
  if (!(PyArray_TYPE(pyarray) == NPY_DOUBLE)) {
    PyErr_SetString(PyExc_TypeError, "Not a NumPy array of
    doubles");
    return NULL; ;
  }
  $1 = int(pyarray->dimensions[0]);
  $2 = (double*)pyarray->data;
}

/* Wrap ODEFieldSolver::get_s in a Python function */
%rename (_get_s) ODEFieldSolver::get_s;

%extend ODEFieldSolver{
  %pythoncode%{
    def get_s(self):
      import numpy as np
      a = np.zeros(self.size())
      self._get_s(a)
      return a
  }
}

#include std_string.i
#include "ODEFieldSolver.h"

```

To make SWIG generate the wrapper code, we run

```
swig -python -c++ -I. -I.. ODEFieldSolver.i
```

SWIG supports many languages and we therefore specify what languages we need wrapper code for by the `-python` and `-c++` flags. Further, `-I` is used to specify where SWIG should look for interface files (with extension `.i`). The C++ class files are located in the parent directory. SWIG will from this command generate two files `ODEFieldSolver.py` and `ODEFieldSolver_wrap.cxx`. The latter needs to be compiled and linked with the `ODEFieldSolver` code to form a shared library with name `_ODEFieldSolver.so`. The `ODEFieldSolver.py` file is the module to use from Python and this is nothing but a Python class wrapper to `_ODEFieldSolver.so` module. Building the shared library is most conveniently done via a standard `setup.py` script. The following [setup.py](#) file provides an appropriate recipe for writing this kind of files:

```

import os, numpy
from distutils.core import setup, Extension
name = 'ODEFieldSolver'
swig_cmd = 'swig -python -c++ -I.. -I. %s.i' % name
os.system(swig_cmd)
sources = ['./%s.cpp' % name, '%s_wrap.cxx' % name]
setup(name=name,
      ext_modules= [
          Extension('_', + name,
                    sources,
                    include_dirs=['..',
                                  numpy.get_include() +
                                  "/numpy"])])

```

To create and install the extension module locally in the current working directory (`.`), we run

```
python setup.py install --install-platlib=.
```

Now we can do `import ODEFieldSolver` in Python and access the C++ class as a Python class.

The FEniCS solver for the parabolic PDE can be implemented as a class:

```

class ParabolicSolver:
    def __init__(self, N, dt):
        """Set up PDE problem for NxN mesh and time step dt."""
        from dolfin import UnitSquareMesh, FunctionSpace,
            TrialFunction, \
            TestFunction, Function, dx, dot, grad

        mesh = UnitSquareMesh(N,N)
        self.V = V = FunctionSpace(mesh, "Lagrange", 1)

        u = TrialFunction(V)
        v = TestFunction(V)

        a = u*v*dx + dt*dot(grad(u), grad(v))*dx

        self.a = a
        self.dt = dt
        self.mesh = mesh
        self.U = Function(V)

    def advance_one_timestep(self, f, u_1):
        """
        Solve the PDE for one time step.
        f: the source term in the PDE.
        u_1: solution at the previous time step.
        """
        from dolfin import TestFunction, dx, solve

        V, a, dt = self.V, self.a, self.dt # strip off self prefix
        v = TestFunction(V)
        L = (u_1 + dt*f)*v*dx

        solve(self.a == L, self.U)

```



```
return self.U
```

The following pseudo code illustrates how to work with this code and the ODE solver:

```
for i in range(num_time_steps): # time loop
    <compute f>
    <call ParabolicSolver's advance_one_time_step>
    <compute g>
    <call the ODE solver>
```

A complete code goes as follows:

```
import dolfin
import numpy

def F(S, U):
    if isinstance(S, dolfin.Function) and isinstance(U,
        dolfin.Function):
        from dolfin import *
        f = sin(S)*exp(U)
        return f
    if isinstance(S, numpy.ndarray) and isinstance(U,
        numpy.ndarray):
        from numpy import *
        f = sin(S)*exp(U)
        return f

class ParabolicSolver:
    def __init__(self, N, dt):
        """Set up PDE problem for NxN mesh and time step dt."""
        from dolfin import UnitSquareMesh, FunctionSpace,
            TrialFunction, \
            TestFunction, Function, dx, dot, grad

        mesh = UnitSquareMesh(N,N)
        self.V = V = FunctionSpace(mesh, "Lagrange", 1)

        u = TrialFunction(V)
        v = TestFunction(V)

        a = u*v*dx + dt*dot(grad(u), grad(v))*dx

        self.a = a
        self.dt = dt
        self.mesh = mesh
        self.U = Function(V)

    def advance_one_timestep(self, f, u_1):
        """
        Solve the PDE for one time step.
        f: the source term in the PDE.
        u_1: solution at the previous time step.
        """
        from dolfin import TestFunction, dx, solve

        V, a, dt = self.V, self.a, self.dt # strip off self prefix
        v = TestFunction(V)
        L = (u_1 + dt*f)*v*dx
```

```

        solve(self.a == L, self.U)
        return self.U

import dolfin
import numpy

N = 12      # mesh partition
dt = 0.01  # time step
parabolicsolver = ParabolicSolver(N, dt)
U1 = dolfin.Function(parabolicsolver.V)
U0 = dolfin.Function(parabolicsolver.V)
U0.vector()[:] = numpy.random.random(parabolicsolver.V.dim())

Q = dolfin.FunctionSpace(parabolicsolver.mesh, "DG", 0)
S0_ex = dolfin.Expression("x[0]")
S0 = dolfin.interpolate(S0_ex, Q)
S1 = dolfin.Function(Q)

import ODEFieldSolver # import module wrapping the ODE solver
odesolver = ODEFieldSolver.ODEFieldSolver()
odesolver.redim(S0.vector().size(0))
odesolver.set_IC(S0.vector().array())
plot = True

for i in range(0, 23): # time loop
    f = F(S0, U0)
    U1 = parabolicsolver.advance_one_timestep(f, U0)

    U1c = dolfin.project(U1, Q)

    odesolver.set_u(U1c.vector().array())
    odesolver.advance_one_timestep()
    S1.vector()[:] = odesolver.get_s()

    U0 = U1
    S0 = S1

    if plot:
        dolfin.plot(U1, title="U")
        dolfin.plot(S1, title="S")
        dolfin.interactive()

```

5.4 Wrapping with Cython

Cython can also be used to wrap C/C++ code. To this end we define the C++ class in a `.pyx` file:

```

cimport numpy as cnp
import numpy as np

cdef extern from "ODEFieldSolver.h":

    cppclass ODEFieldSolver_ "ODEFieldSolver":
        ODEFieldSolver()
        void redim(int n)
        int size()

```

```

double g(double s, double u)
void set_dt(double dt)
void set_IC(int n, double* array)
void set_const_IC(double s0)
void set_u (int n, double* array)
void set_IC(double val)
void set_u (double const_value)
void advance_one_timestep()
void get_s (int n, double* array)

```

Here, we redefine the name to `ODEFieldSolver_` such that we may shadow the underlying class with the following Python class:

```

cdef class ODEFieldSolver:
    cdef ODEFieldSolver_ *wrapped

    def __init__(self):
        self.wrapped = new ODEFieldSolver_()

    def __dealloc__(self):
        if self.wrapped != NULL:
            del self.wrapped

```

We have a pointer called `wrapped` to the underlying SWIG-generated C interface to the C++ code. Simple functions like e.g. `redim` are straightforward to wrap,

```

def redim(self, n):
    self.wrapped.redim(n)

```

For the `set_IC` function we need to check that the input argument is a contiguous 1-dimensional numpy array of type double. This is specified as `cnp.ndarray[double, ndim=1, mode='c']`. Further, we check that the length of the input array is the same as `self.wrapped.size` before we pass the input array to the underlying C++ object.

```

def set_IC(self, cnp.ndarray[double, ndim=1, mode='c']
array):
    if array.shape[0] != self.wrapped.size():
        raise ValueError('incorrect dimension on array')
    self.wrapped.set_IC(array.shape[0], &array[0])

```

We allow the user to employ the `get_s` function both with and without an input argument, referred to as `out`. If the user does not supply any input, i.e., if `out` is `None` then we create an array of appropriate type and size. Otherwise, we check that `out` has the appropriate type and size before the array is passed to the C++ object's `get_s` function. At the end, we simply return `out`.

```

def get_s(self,
cnp.ndarray[double, ndim=1, mode='c'] out=None):
    if out is None:
        out = np.empty(self.wrapped.size(), dtype=np.double)
    elif out.shape[0] != self.wrapped.size():
        raise ValueError('incorrect dimension on out')
    self.wrapped.get_s(out.shape[0], &out[0])

```

```
return out
```

Finally, the module is built using a `setup.py` script for mixing Python, Cython, and C++ code:

```
from distutils.core import setup
from distutils.extension import Extension
from Cython.Distutils import build_ext

setup(
    cmdclass = {'build_ext': build_ext},
    ext_modules = [Extension("ODEFieldSolver",
                             ["ODEFieldSolver.pyx",
                              "../ODEFieldSolver.cpp"],
                             include_dirs=['..'],
                             language='c++')]
)
```

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